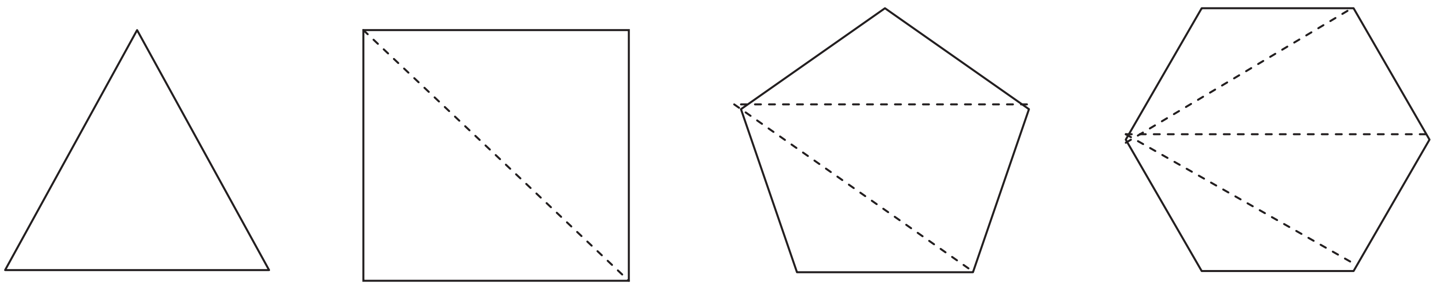
Regular Polygons and Angle Relationships KEY

Part I
A *regular* polygon is a special polygon which is both equilateral and equiangular. Suppose each of the polygons below is a *regular* polygon, and is divided into triangular regions as shown.



1. What is the interior angle sum for the triangle? What theorem justifies this conjecture?
   180°. Triangle Angle Sum Theorem.

2. Based the Triangle Angle Sum Theorem, what can be concluded about the interior angle sum of the regular quadrilateral? Explain.
   360°. Since there are two triangular regions, the interior angle sum is 180°+180° or 360°.

3. Based the Triangle Angle Sum Theorem, what can be concluded about the interior angle sum of the regular pentagon? Explain.
   540°. Since there are three triangular regions, the interior angle sum is 180°+180°+180°

4. Use this reasoning to complete the table below.

Polygon Name	Number of Sides	Number of Triangular Regions	Process to Find the Sum of the Interior Angles	Sum of the Interior Angles
Triangle	**3**	1	(1)180	**180°**
Quadrilateral	4	2	(2)180	360°
Pentagon	5	3	(3)180	540°
Hexagon	6	4	(4)180	720°
***n*-gon**	n	n - 2	(n – 2)180	180n - 360

5. Based on the data in the table, describe how the angle sum changes as additional sides are added to the polygon?
   Each time an additional side is added to the polygon, the interior angle sum increases by 180°.

6. Based on the previous answer, what type of algebraic relationship occurs between the interior angle sum and the number of sides of the polygon?
   Since the rate of change of the interior angle sum is constant, the relationship between the interior angle sum and the number of sides of the polygon is linear.

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17. Repeat the procedure to find the measure of each of the interior and exterior angles of a regular pentagon, regular hexagon, regular heptagon, and regular octagon as well as the exterior angle sum. Record your data in the table below.

Polygon Name	Number of Sides, n	Sum of the Interior Angles $(n-2)180$	Process to Find the Measure of an Interior Angle	Measure of Each Interior Angle	Measure of Each Exterior Angle	Exterior Angle Sum (one angle at each vertex)
Triangle	3	180^o	180/3	60°	120°	360°
Quadrilateral	4	360^o	360/4	90°	90°	360°
Pentagon	5	540^o	540/5	108°	72°	360°
Hexagon	6	720^o	720/6	120°	60°	360°
Heptagon	7	900^o	900/7	128.57°	51.43°	360°
Octagon	8	1080^o	1080/8	135°	45°	360°
n-gon	n	$180n - 360$	$\frac{(n-2)180}{n}$	$\frac{(n-2)180}{n}$	$180 - \frac{(n-2)180}{n}$	360°

a. Describe the process for finding the measure of an interior angle of a regular polygon.
Divide the interior angle sum of the polygon by the number of sides of the polygon (which is also the number of interior angles of the polygon).

b. Describe the process for finding the measure of an exterior angle of a regular polygon.
Since the interior and exterior angles form a linear pair, they are supplementary. Therefore, the measure of the exterior angle can be found by subtracting the measure of the interior angle from 180.